Identification of Scalar Meson Field with Perfect Fluid in Bimetric Theory for a Spherically Symmetric Cosmological Model

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Abstract – The eigen values of scalar meson field and perfect fluid for a spherically symmetric homogeneous metric in Bimetric theory has been compared. It is found that the scalar meson field does not represent the perfect fluid for the space time. However the equivalency of these two fields is true in general theory of relativity in Bimetric theory. Some physical and geometrical properties of both the models are also discussed.

Index Terms – Scalar Meson Field, Perfect Fluid, Bimetric Theory, Spherically Symmetric.

1. INTRODUCTION

Rosen [1] has proposed a new theory of gravitation, which is known as Bimetric theory of gravitation. It is based on a simple form of Lagrangian and has simpler mathematical structure than that of general theory of relativity. It also satisfies the covariance and equivalence principles. This theory makes use of Riemannian metric tensor g_{ij} and associated flat space metric tensor γ_{ij} describing the properties of space time while g_{ij} describes the gravitational potential tensor which determines the interaction between matter and gravitation. Rosen [1], Mohanty and Sahoo [2], Mohanty and Pradhan [3], Reddy and Venkateswaralu [4], Israelit [5], Karde and Dhoble[6], J.Anderson[7], Tabenbsky and Taub[8], Tiwari, Rao and Mohanty [9] have studied several aspects of this theory.

In this paper, we have studied that the micro cosmological model representing scalar meson field does not survive for stiff fluid model representing perfect fluid distribution in Bimetric relativity. The field equation of both the models are solved and the consequences of the results through different physical quantities involved in the solution are analyzed.

2. FIELD EQUATION

The field equation of Bimetric theory of gravitation formulated by Rosen [1] are

$$N_{ij} - \frac{1}{2}N g_{ij} = -8\pi K T_{ij} \tag{1}$$

Where $N = N_i^j = \frac{1}{2}\gamma^{ab}(g^{hi} g_{hi}|a)|b$

Where $K = \sqrt{\frac{g}{\gamma}}$ with $g = \det g_{ij}$ and $\gamma = \det \gamma_{ij}$. Here a vertical bar '|' stands for covariant differentiation with respect to γ_{ij} and T_{ij} is the energy momentum of matter field.

3. PERFECT FLUID

We consider the spherically symmetric line element of the form

$$dS^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta\, d\phi^2 + e^{\mu} dt^2 \tag{2}$$

Where 'λ', 'μ' are functions of 't' only. The background flat space-time corresponding to the metric (2) is

$$d\sigma^2 = -dr^2 - d\theta^2 - d\phi^2 + dt^2 \tag{3}$$

The energy momentum tensor for perfect fluid is given by

$$T_{ij} = (P + \rho)\, U_i\, U_j - P g_{ij} \tag{4}$$

Together with $g_{ij}\, U^i\, U^j = 1$, where U^i is the four velocity vector of the fluid distribution having 'P' and 'ρ' as the proper pressure and energy density of the fluid respectively.

Using co-moving co-ordinate system the field equation (1) for the metrics (2) and (3) corresponding to the energy momentum tensor (4) in Bimetric theory can be written explicitly as

$$\lambda_{44} - \mu_{44} = 32\pi K P \tag{5}$$

$$\lambda_{44} + \mu_{44} = -32\pi K P \tag{6}$$

$$\lambda_{44} - \mu_{44} = 32\pi K \rho \tag{7}$$

$$\text{where } K = e^{\frac{\lambda+\mu}{2}} r^2 sin\theta \tag{8}$$

The subscript '4' after field variable stands for ordinary differentiation with respect to co-ordinate 't'.

For radiating model ($\rho = 3P$):

Using equations (6) and (7) ,we get the value of 'λ_{44}' as

$$\lambda_{44} = 32\pi KP.$$

Using equation (8), we get

$$\lambda_{44} = 32\pi Pr^2 \, sin\theta \, e^{\frac{\lambda+\mu}{2}} \quad (9)$$

To avoid the complicacy of integration while finding the value of 'λ', we have taken

$$\lambda = -\mu \quad . \quad (10)$$

So that the value of 'λ' is found to be

$$\lambda = 16\pi Pr^2 \, sin\theta \, t^2 + c_2 t + c_3 \quad (11)$$

By using equation (9), we get

$$\mu = -(16\pi Pr^2 \, sin\theta \, t^2 + c_2 t + c_3) \quad (12)$$

Using equations (11) and (12) in metric (2),the metric is designed as

$$dS^2 = -e^{(16\pi Pr^2 \, sin\theta \, t^2 + c_2 t + c_3)} \, dr^2 - r^2 \, d\theta^2 - r^2 \, sin^2\theta \, d\phi^2 + e^{-(16\pi Pr^2 \, sin\theta \, t^2 + c_2 t + c_3)} \, dt^2 \quad (13)$$

This model expands with respect to parameter 'r' (i.e. Volume $\propto r^2$) . But the expansion does not depend on cosmic time. Also the model shows anisotropy throughout the evolution as $\sigma^2 \neq 0$.

4. MASSIVE SCALAR FIELD

In this section, we consider the region of the space-time with attractive massive scalar meson field whose energy momentum tensor is given by

$$T_{ij} = V_{;i} \, V_{;j} - \frac{1}{2} g_{ij} (V_{;m} \, V^{;m} - M^2 \, V^2) \quad (14)$$

together with

$$g^{ij} \, V_{;ij} + M^2 \, V = 0 \quad , \quad (15)$$

Where 'M' is the mass parameter of the scalar meson field 'V'. Here after words the subscript comma and semicolon after a field variable represent ordinary and covariant differentiation with respect to 't' and 'g_{ij}' respectively.

The explicit form of the field equation (1) for metric (2) and (3) with energy momentum tensor (13) are obtained as

$$\lambda_{44} - \mu_{44} = 32\pi K \left(\frac{V_4^2}{8} + \frac{M^2 \, V^2}{2} \right) \quad (16)$$

$$\lambda_{44} + \mu_{44} = 32\pi K \left(\frac{V_4^2}{8} + \frac{M^2 \, V^2}{2} \right) \quad (17)$$

$$\lambda_{44} - \mu_{44} = 32\pi K \left[\frac{e^{\mu}}{4} \, V_4^2 + \left(\frac{V_4^2}{8} + \frac{M^2 \, V^2}{2} \right) \right] \quad (18)$$

From equations (16) and (18), we get

$$\frac{e^{\mu}}{4} \, V_4^2 = 0,$$

which implies either $e^{\mu} = 0$ or $V_4^2 = 0$.

If $e^{\mu} = 0$, metric (2) reduces to three dimensional space. Hence taking $V_4^2 = 0$, we get V = constant = c_4 .

Adding equations (16) and (17), we get

$$\lambda_{44} = 32\pi K \left(\frac{V_4^2}{8} + \frac{M^2 \, V^2}{2} \right)$$

Putting $V_4^2 = 0$ and $V = c_4$, the above equation reduces to

$$\lambda_{44} = 16\pi K M^2 \, c_4^2.$$

Using equation (8),we get

$$\lambda_{44} = 16\pi M^2 \, c_4^2 \, r^2 \, sin\theta \, e^{\frac{\lambda+\mu}{2}}.$$

Using equation (9) and solving we get the value of 'λ' as

$$\lambda = 8\pi M^2 \, c_4^2 \, r^2 \, sin\theta \, t^2 + c_5 t + c_6 \quad (19)$$

Ultimately

$$\mu = -(8\pi M^2 \, c_4^2 \, r^2 \, sin\theta \, t^2 + c_5 t + c_6) \quad . \quad (20)$$

So metric (2) takes the form

$$dS^2 = -e^{(8\pi M^2 c_4^2 r^2 sin\theta \, t^2 + c_5 + c_6)} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta \, d\phi^2 + e^{-(8\pi M^2 c_4^2 r^2 sin\theta \, t^2 + c_5 + c_6)} dt^2 \quad (21)$$

The Klein-Gorden equation (14) for metric (2) becomes

$$2\mu_{44+}\mu_4 + 4M^2 \, V \, e^{-\frac{\mu}{2}} = 0.$$

Using equations (19), (20), putting $V = c_4$, $\theta = 90^0$ and solving ,we get the value of 't' as

$$t = \frac{e^{8\pi M^2 \, c_4^2 \, r^2}}{4\pi c_4 \, r^2} - \frac{c_5}{16\pi M^2 \, c_4^2 \, r^2} - 2 \quad (22)$$

This condition is highly essential for existance of scalar meson field given by equation (13).

From equation (22) ,we observe that with increase in the mass factor of scalar meson field the time factor of our metric increases.

Expansion of the model is independent of time, but proportional to 'r^2'. The model shows anisotropy throughout the evolution. The energy density associated with the scalar meson field

$$\omega = \frac{1}{2}(V_4^2 + M^2 V^2) \quad (23)$$

becomes $\omega = \frac{M^2 c_4^2}{2}$ (24)

i.e. $\omega \propto M^2$

5. IDENTIFICATION OF SCALAR FIELD WITH PERFECT FLUID

The eigen values λ_i $(i = 1, 2, 3, 4)$ of the energy momentum tensor T_j^i are given by the determinantal equation

$$\left| T_j^i - \lambda\, \delta_j^i \right| = 0. \quad (25)$$

For perfect fluid, the eigen values given by the equation are

$$\lambda_1 = \lambda_2 = \lambda_3 = -P \text{ and } \lambda_4 = \rho\,, \quad (26)$$

Whereas for massive scalar field given by equation (13) are obtained as

$$T_{11} = T_{22} = T_{33} = -\left(\frac{V_4^2}{8} + \frac{M^2 V^2}{2}\right) \quad \text{and}$$

$$T_{44} = \frac{e^{\mu}}{4} V_4^2 + \left(\frac{V_4^2}{8} + \frac{M^2 V^2}{2}\right). \quad (27)$$

Comparing equation (26) and (27), we get

$$P = \frac{V_4^2}{8} + \frac{M^2 V^2}{2}, \quad (28)$$

$$\rho = \frac{e^{\mu}}{4} V_4^2 + \left(\frac{V_4^2}{8} + \frac{M^2 V^2}{2}\right) = \frac{e^{\mu}}{4} V_4^2, \quad (29)$$

Which ,for radiating model $(\rho = 3P)$, is true only when

$$\frac{e^{\mu}}{4} V_4^2 = 2P$$

or $P = \frac{e^{\mu}}{8} V_4^2$. (30)

Comparing equation (28) and (30), we get

$$\frac{e^{\mu}}{8} V_4^2 = \frac{V_4^2}{8} + \frac{M^2 V^2}{2},$$

which implies that $\mu = 0$ and $M = 0$ or $V = 0$.

Thus the scalar meson field is absent for radiating model of irrotational perfect fluid for space time (2) in Bimetric theory. However the equivalence of these two fields is true in general theory of relativity.

6. CONCLUSION

In this paper, it has shown that in Bimetric theory the micro cosmological model representing scalar meson field does not survive for stiff fluid model representing perfect fluid distribution for the metric(2), because $M = 0$ or $V = 0$. It is seen that in both the cases expansion of the universe is independent of 't' but proportional to 'r^2' and the models show anisotropy throughout the evolution.

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